## PHYSICS 211

## Final

## TIME: 120 minutes

January 14, 2013

DO NOT OPEN THIS EXAM BEFORE YOU ARE TOLD TO BEGIN

NAME $\qquad$

ID Number $\qquad$

Grading

| 1 |  |
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| TOTAL |  |

$\qquad$Check if solution is continued on the back.

1. (30) We consider an infinitely long wire with a diameter $d$ and length $l$.
a. (8) By applying Gauss's law, determine the electric field at a distance $r>d$ from the wire when the wire is charged with $Q$.
b. (4) Determine the capacitance of the wire at $r$ (in this case, $C$ will depend on $r>d$ )
c. (8) By Applying Ampere's law, determine the magnetic field at the same distance $r>d$ from the wire when there is a current flowing in the $+z$-direction.
d. (6) At the position $r>d$, a proton moves in the $+z$-direction parallel to the wire, determine its velocity assuming it is constant in the case where both $Q(>0)$ and $I$ are present.
e. (4) Any modifications if the proton is replaced by an electron? Justify briefly.
$\qquad$Check if solution is continued on the back.
2. (12) A solenoid, with length $l$ and area $A$ is connected to an electromotive force $\varepsilon$.
a. (6) If the magnetic field inside the solenoid has the expression $B=\mu_{0} n I$, where $n$ is the number of turns per unit length, find the expression of its inductance.
b. (6) When the switch closes, hence the current is rising, show that the inductance of the solenoid can be written as $L=-\varepsilon /(d i / d t)$ where $i$ is the instantaneous current.
3. (27) Consider the circuit below with the three resistors, $R_{1}, R_{2}$ and $R_{3}$ and an electromotive force $\varepsilon$ and an inductor $L$.(Hint: the solution of the differential equation $d x / d t+\alpha x=\beta$ is $x=\beta / \alpha\left(1-e^{-\alpha t}\right)$.

a. (14) As a function of time, just after closing the switch, determine the expression of the current $I_{3}$ in $R_{3}$ and $I_{1}$ in $R_{1}$.
$\qquad$Check if solution is continued on the back.
b. (6) As time $t$ tends to infinity, find the expression of the currents in the three resistors.
c. (7) Compare the power generated to the power dissipated as $t$ tends to infinity. Take $\varepsilon=10 \mathrm{~V}, \mathrm{~L}=1 \mathrm{H}, R_{1}=4 \Omega, R_{2}=4 \Omega$ and $R_{3}=8 \Omega$.
d. (4) Determine the energy generated by the battery at an arbitrary time $t_{0}$. And comment on the different contributions.
4. (27) We wish to study a circuit composed of an AC power source, supplying a current $I=I_{\mathrm{m}} \sin (\omega t)$, a resistor $R$ and an inductor $L$.
(a) (4) First, consider that the resistance alone is connected to the power supply, determine the expression of the potential difference across it.
(b) (4) Consider that the inductor alone is connected to the power supply, determine the expression of the potential difference across it.
$\qquad$Check if solution is continued on the back.
(c)(6) When both $R$ and $L$ are put in the series, determine the impedance (denoted be $Z$ or $X$ ) of the circuit.
(d)(3) Determine the expression of the circuit phase.
(e)(4) Discuss the behavior of this circuit of low frequencies $(\omega \rightarrow 0)$ and for $(\omega \rightarrow \infty)$ and plot the current $I_{\text {max }}$ vs. $\omega$


We insert a capacitor in parallel to the inductor as shown in the figure below and the AC power source produces now a potential difference $\varepsilon=\varepsilon_{0} \cos (\omega t)$.

(f)(3) What are the maximum values of $I_{\mathrm{L}}, I_{\mathrm{c}}$ and $I_{\mathrm{R}}$ in the case where $\omega \rightarrow 0$
(g)(3) What are the maximum values of $I_{\mathrm{L}}, I_{\mathrm{c}}$ and $I_{\mathrm{R}}$ in the case where $\omega \rightarrow \infty$

## SCRATCH PAPER

Nothing on this page will be graded

